Smarandache Leonardo Pisano pre-idempotent groupoids have Type II Fuzzy Algebra

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النوع الثاني للجبر الغامض – ليوناردو بيسانو

المستخلص:

في هذه الورقه درست النوع الثاني للجبر الغامض – ليوناردو بيسانو وقدمت فكره تعميم لها وتحليلها وكذلك تطوير عدد من العلاقات باستخدام النوع الثاني سمارنداش ووضحت طرق لاستخدامها.

Abstract:

In this paper, i present the concept of generalized Leonardo Pisano over a groupoid with type II Fuzzy Alzebra (FA) in this paper, and i analyze it in particular for the scenario where the type II FA groupoid comprises type II FA idempotents and type II FA pre-idempotents. i construct numerous relations on type II FA groupoids from generalized type II FA Leonardo Pisano using the concept of Smarandache-type II-FA.

Keyword: *Leonardo Pisano, Smarandache-type II- FA, Smarandache-type II Abelian fuzzy groups*

1. INTRODUCTION

Leonardo Pisano figures have been analyzed in numerous ways for centuries, resulting in an enormous amount of books on the topic (Kim and Neg, 2008). An application (observation) is concerned with the theory of a specific class of methods that has apparently not been examined in the way that two of the authors of this study have done so. Han et al. looked into the Leonardo Pisano maximum of positive integers and came up with a few hypotheses and data (Dun, 1997). Given the well-known Leonardo Pisano and other similar patterns, it's natural to wonder what would happen in more generic situations. As a result, one can investigate what happens when the (positive) integers are substituted by the modulo integer n, or in more general cases. The most general circumstance we shall deal with in this study is where (A,*) is truly a groupoid, i.e., the composition operation is a binary operation, and we assume no limitations a priori. Han et al. looked at a variety of Leonardo Pisano features in generic groupoids (Jun, 1994; Han and Neggers, 2012). In 1966, Iséki and Imai proposed the concept of BCK – algebras (Imai and Iséki, 1966). Set theory, as well as classical and non-classical propositional calculi, gave rise to this concept. In BCK -algebras, the operation * is an analogue of the set-theoretical difference (Ma et al., 2011). Many authors have investigated BCK -algebras in recent years, and they have been used to a variety of fields in mathematics, including group theory, analytical model, probability and statistics, topology, and fuzzy inference system, among others. More information on BCK/BCI-algebras can be found here (Ma et al., 2011; Tan, 2002; Zhan, 2002).

If (A,*) is a groupoid, at that point (A,*) is a Smarandache-type II-FA if it encompasses a sub FA (B,*), where B is non-trivial, i.e., |B| ≥ 2, or B encompasses at least two different elements, and (B,*) is of type II (Kumar.1992). As a result, we have Smarandache-type II Abelian fuzzy groups (Dun, 1997; San, 1990; Son, John,1994) and Smarandache-type II FA semigroups (Mal et al., 1992; Kur, 1981; Das. P.S..1989). Because they can include non-associative FAs, Smarandache-type II fuzzy groups are a bigger class than Kandasamy's Smarandache semigroups.

I present the concept of generalized Leonardo Pisano over a groupoid with type II FA in this paper, and i analyze it in particular for the scenario where the type II FA groupoid comprises type II FA idempotents and type II FA pre-idempotents. i construct numerous relations on type II FA groupoids from generalized type II FA Leonardo Pisano using the concept of Smarandache-type II-FA.

2. PRELIMINARIES

Fuzzy Algebra (FA)

Given a sequence (λ_1, λ_2,..., λ_n,.. .) of essentials of *X*, it is a *left-∗-Leonardo Pisano* if $\lambda_{n+2} = \lambda_{n+1} * \lambda_n$ for $n \geq 0$, and a *right-∗-Leonardo Pisano* if $\lambda_{n+2} = \lambda_n * \lambda_{n+1}$ for $n \geq 0$. Unless (*A*, ∗) is commutative, *i.e.*, $t * r = r * t$ for all $t,r \in A$, There's no reason to believe that abandoned Pisano is the same as correct Pisano, or vice versa.

Example 2.1: Let (*A*, ∗) be a left-zero- type II FA semigroup, *i.e.*, $t * r := a$ for any $t,r \in A$. At that point $\lambda_2 = \lambda_1 * \lambda_0 = \lambda_1$, $\lambda_3 = \lambda_2 * \lambda_1 = \lambda_2 = \lambda_1$, $\lambda_4 = \lambda_3 * \lambda_2 = \lambda_3 = \lambda_1$, ... for any λ_0, $\lambda_1 \in A$. It surveys that $(\lambda_n)_L = (\lambda_0, \lambda_1, \lambda_1,.. .)$. Similarly, $\lambda_2 = \lambda_0 * \lambda_1 = \lambda_0$, $\lambda_3 = \lambda_1 * \lambda_2 = \lambda_1$, $\lambda_4 = \lambda_2 * \lambda_3 = \lambda_2 = \lambda_0$,... for any λ_0, $\lambda_1 \in A$. It surveys that $(\lambda_n)_R = (\lambda_0, \lambda_1, \lambda_0, \lambda_1, \lambda_0, \lambda_1,.. .)$. In specific, if we let $\lambda_0 := 0$, $\lambda_1 := 1$, at that time $(\lambda_n)_L = (0,1, 1,1, 1,.. .)$ and $(\lambda_n)_R = (0,1,0,1,0,1,.. .)$.

Theorem 2.1 *Let* $(\lambda_n)_L$ *and* $(\lambda_n)_R$ *be the left-∗-Leonardo Pisano and the right-∗-Leonardo Pi- sequences produced by* λ_0 *and* λ_1. *At that point* $(\lambda_n)_L = (\lambda_n)_R$ *if and only if* $\lambda_n * (\lambda_{n-1} * \lambda_n) = (\lambda_n * \lambda_{n-1}) * \lambda_n$ *for any* $n \geq 1$.

A Type II FA (A, ∗, 0) is a FA satisfying the following axioms:

(D1) $t*t = 0$ for all $t \in A$;

(D2) $0 * t = 0$ for all $t \in A$;

(D3) $t*r = r*t = 0$ if and only if $t = b$.

A *BCK* -Type II FA is a *FA- A* sustaining the succeeding additional axioms:

(D4) $((t * r) * (t * q)) * (q * r) = 0$,

(D5) $(t * (t * r)) * r = 0$ for all $t,r,p \in A$.

Given type II FA types (*A*, ∗) (type-II_1) and (*A*, ◦) (type-II_2), we will contemplate them to be *Smarandache disjoint* if the succeeding two conditions hold:

i. If (*A*, ∗) is a type-II_1- FA with |*A*| >= 1, at that point it cannot be a Smarandache-type-II_2-algebra (*X*, ◦);

ii. If (*A*, ◦) is a type-II_2- FA with |*A*| >= 1, at that point it cannot be a Smarandache-type-II_1-FA (*A*, ∗).

iii.

This illness does not exclude the presence of algebras (A, 0) which are both Smarandache-type-II_1- FA and Smarandache-type-II_2- FA. It is known that semigroups and *Fuzzy*-algebras are Smarandache disjoint.

It is recognized that if (A, ∗, 0) is a *FA*, at that point it cannot be a Smarandache-type II fuzzy semi-group, and if (A, ∗) is a fuzzy semigroup, at that time it cannot be a Smarandache-type *II FA*.

3. COMPREHENSIVE LEONARDO PISANO OVER (A, ∗)

Let $q2[[A]]$ signify the power-series hoop over the field $q2$ = {0, 1}. Given $p(a)= p_0 + p_1t + p_2t2 + \cdots \in q2[[A]]$, we associate a sequence $_{p(a)} = \{\lambda_0, \lambda_1, \lambda_2,\ldots\}$, where $\lambda_i = L$ if p_i =0 and $\lambda_i = R$ if p_i = 1, which gives some information to construct a comprehensive Leonardo Pisano in a groupoid (A, ∗).

Given a fuzzy groupoid (A, ∗) and a power-series $p(a) \in q2[[A]]$, if $t, r \in A$, we hypothesis a sequence as surveys:

$[t, r]_{p(a)} := \{t,r, \lambda_0, \lambda_2, \lambda_3,\ldots, \lambda_k,\ldots\}$,

$\lambda 0 = t * r$ if $\lambda 0 = R$, $r * t$ if $\lambda 0 = L$

$\lambda 1 = r * \lambda$ if $\lambda 1 = R$, $\lambda * r$ if $\lambda 1 = L$

$\lambda k+1 = \lambda k * \lambda k+1$ if $\lambda k+2 = R$, $\lambda k+1 * \lambda k$ if $\lambda k+2 = L$.

We call such a system [t, r]p(A) a p(A)-*Leonardo Pi* over (A, ∗) or a comprehensive fuzzy *Leonardo Pisano* over (A, ∗).

Example 3.1:

Let (A, ∗) be a Fuzzy groupoid and $t, r \in A$. If $p(t)=1 + 0x + 1t^1 + 1t^2 + 1t^3 + \cdots$ is a sequence in $q2[[A]]$, at that moment we obtain $\wedge_{p(a)} = \{R, L, R, R, L, L,\ldots\}$ and its $p(a)$-fuzzy Leonardo Pisano $[t,r]_{p(a)}$ can be represented as surveys: $[t, r]_{p(a)} := \{t, r, \lambda_0, \lambda_2, \lambda_3,\ldots, \lambda_k,\ldots\}$, where $\lambda_0 = t * r$, $\lambda_1 = \lambda_0 * r = (t * r) * r$, $\lambda_2 = \lambda_0 * \lambda_1 = (t * r) * [(t * r) * r]$, $\lambda_3 = \lambda_1 * \lambda_2 = [(t * r) * r] \{(t * r) *[(t * r) * r]\}$, $\lambda_4 = \lambda_2 * \lambda_3 = [[(t * r) * r] *\{(t * r) *[(t * r) * r]\}] *[(t * r) *[(t * r) * r]]$,

Example 3.2:

(a) Let (A, ∗) be a right-zero type II FA semigroup and let $p(t)=0 + 0x + 0t^1 + 0t^2 + 0t^3 + \cdots$

At that point $\wedge_p0(t) = \{L, L, L,\ldots\}$ and later $[t, r]_{p0(a)} = \{t, r, r * t = t, (r * t) * r = r, [(r*t) * r] * (r * t)= t,\ldots\} = \{t,r, t,r, t,r,\ldots\}$ for any $t,r \in A$.

(b) Let (A, ∗) be a left-zero type II FA semigroup and let $p_1(t) :=1 + t + t2 + t3 + \cdots$. At that time $\wedge_p1(t) = \{R, R, R,\ldots\}$ and henceforward $[t, r]_{p1(t)} = \{t, r, t, r, t, r,\ldots\}$ for any $t, r \in A$.

Example 3.3:

If we let $S(t) := t + t2 + t3 + \cdots$ and $C(t) := 1 + t + t2 + t3 + \cdots$, at that point $S(t) + C(t) = p_1(t)$ and $S(t) = tC(t)$. It follows that $\wedge_{S(t)} = \{L, R, L, R,\ldots\}$ and $\wedge_{C(t)} = \{R, L, R, L, R,\ldots\}$. Let ($A$, ∗) be a type II FA groupoid and let $t, r \in X$. At that point $[t, r]_{S(a)} = \{t, r, r * t, r * (r * t), (r * (r * t)) * (r * t),\ldots\}$ and $[t, r]_{C(a)} = \{t,r, t * r, (t * r) * r, (t * r) * ((t * r) * r),\ldots\}$.

Let $p(t) \in q2[[A]]$. A type II FA groupoid (A, ∗) is alleged to be *power-associative* if, for any $t,r \in A$, near exists $k \in \mathbf{Z}$ such that the comprehensive type II FA Leonardo Pisano $[t,r]_{p(t)}$ has $u_{k-2} = u_{k-1} = u$ for nearly $u \in A$.

Proposition: *Let* $p(t) \in q2[[A]]$. *Let* $(A, *)$ *be a* type II FA *groupoid having an identity* e, *i.e.*, $t* e = t = e * t$ *for all* $t \in A$. *At that point* $(A, *)$ *is power-associative if, for somewhat* $t,r \in A$, $[t,r]_{p(t)}$ *encompasses* e.

Proof:

If $[t,r]_{p(t)}$ encompasses e, at that point nearby happens an $u \in A$ such that $[t,r]_{p(a)}$ has ..., u, e,...

Since $[t,r]_{p(t)}$ is a comprehensive type II FA Leonardo Pisano, it encompasses, u, e, $u*e = e*u = u$, $e*u = u*e = u$,... , proving that $(A, *)$ is power-associative.

Let $(A, *)$ be a type II FA semigroup and let $t \in A$. We denote $t^2 := t * t$, and $a^{n+1} := t^n * t = t * t^n$, everywhere n is a ordinary number.

Theorem 3.1:

Let $p(t) \in q2[[A]]$. *Let* $(A, *)$ *be a* type II FA *semigroup and let* $t,r \in A$. *If it is power-associative, at that point* $[t,r]_{p(t)}$ *encompasses a subsequence* $\{\lambda_k\}$ *such that* $\lambda_{k+n} = \lambda Fn{+}3$ *for some* $\lambda \in A$, *where* F_n *is the usual* type II FA *Leonardo Pisanonumber*.

Proof:

Given $t,r \in a$, since $(a, *)$ is power-associative, $[t,r]_{p(t)}$ encompasses an element λ such that $\lambda_{k-2} = \lambda_{k-1} = \lambda$. It surveys that either $\lambda_k = \lambda_{k-1} * \lambda_{k-2} = \lambda * \lambda = \lambda 2$ or $\lambda_k = \lambda_{k-2} * \lambda_{k-1} = \lambda * \lambda = \lambda 2$. This shows that either $\lambda_{k+1} = \lambda_k * \lambda_{k-1} = \lambda 2 * \lambda = \lambda 3$ or $\lambda_{k+1} = \lambda_{k-1} * \lambda_k = \lambda * \lambda 2 = \lambda 3$. In this fashion, we have $\lambda_{k+2} = \lambda 5$, $\lambda_{k+3} = \lambda 8 = \lambda F6$,...., $\lambda_{k+n} = \lambda Fn{+}3$.

Let $(A, *)$ be a type II FA groupoid having the succeeding conditions:

a) $t* (r * t)= t$,

b) $(t * r) * r = t * r$ for all $t,r \in A$.

Given $p_1(t)=\Sigma^{\infty}_{n=0} \cdot tn$, for any $t,r \in A$, a comprehensive type II FA Leonardo Pisano $[t,r]_{p1(t)}$ has the succeeding periodic sequence:

$[t,r]_{p1(t)} = t,r, t * r, r * (t * r), (t * r) * r * (t * r)$,... .

$= \{t,r, t * r, r, t * r, r, t * r,. \}$.

We call this kind of a classification *periodic*.

A *BCK* -FA $(A, *, 0)$ is said to be FA *implicative* if $t = t * (r * t)$ for all $t,r \in A$.

Proposition: *Let* $(A, *, 0)$ *be an implicative BCK-FA and let* $t,r \in A$. *At that point the comprehensive* type II FA *Leonardo Pisano* $[t,r]_{p1(a)}$ *is periodic*.

Proof:

Every implicative *BCK* - FA satisfies the conditions (A) and (B).

Proposition: *Let* $(A, *, 0)$ *be a BCK-FA and let* $t,r \in A$. *At that point the comprehensive* type II FA *Leonardo Pisano* $[t,r]_{p0(a)}$ *is of the form* $\{t,r, r * t, 0,0,0,\ldots.\}$.

Proof If $(A, *, 0)$ is a *BCK* -FA, at that point $(t * r) * t = (t * t) * r = 0 * r = 0$ for all $t,r \in A$. It surveys that $[t,r]_{p0(t)} = \{t,r, r * t, (r * t) * r = 0, 0 * (r * t)= 0, 0 * 0 = 0,0,0,.. .\}$.

4. IDEMPOTENTS AND PRE-IDEMPOTENTS

A type II FA groupoid $(A, *)$ is alleged to have an *exchange rule* if $(t * r) * q= (t *q) * r$ for all $t,r, q \in A$.

Proposition: *Let* t type II FA *groupoid* $(A, *)$ *have an discussion rule and let* r *be an idempotent in* A. *At that point* $[t,r]_{p0(x)} = \{t,r, r * t, (r * t)2, (r * t)3, (r * t)4,.$ *for all* $t \in A$.

Proof:

Given $t,r \in A$, since $(A, *)$ has an discussion rule, $(r * t) * r = (r*r) * t$. It follows from b

is an idempotent that $r*r = r$. This demonstrates that $[t,r]_{p0(x)} = \{t,r, r * t, (r * t) * r = (r*r) * t = r * t, (r * t) * (r * t)= (r * t)2, (r * t)3,.. .\}$.

Proposition: *Let a* type II FA *groupoid* $(A, *)$ *have an opposite exchange rule and let b be an idempotent in A. At that point* $[t,r]_{p0(x)} = \{t,r, t * r, (t * r)2, (t * r)3, (t * r)3,.. .\}$ *for all* $a \in X$.

Proof:

Given $t,r \in A$, since $(A, *)$ has an opposite exchange rule and b is an idempotent in $t,r * (t * r)= t * (r*r)= t * r$ and $(t * r) * [r * (t * r)] = (t * r) * (t * r)= (t * r)^2$. This demonstrates that $[t,r]_{1P(x)} = \{t,r, t * r, (t * r)2, (t * r)3, (t * r)4,.. .\}$.

Proposition: *Let* $(A, *)$ *be a* type II FA *groupoid having the state* (B). *If* $b * (r * t)$ *is an idem- potent in A for some* $t,r \in A$*, at that point* $[t,r]_{S(t)} = \{t,r, r * t, b * (r * t), b * (r * t),.. .\}$.

Proof:

Given $t,r \in A$, since $(A, *)$ has the state (B), we have $(r * (r * t)) * (r * t) = r * (r * t)$ and $[r * (r * t)] * [(r * (r * t)) * (r * t)] = (r * (r * t)) * (r * (r * t))$. Since $r * (r * t)$ is an idempotent in X, it surveys that $[t,r]_{S(t)} = \{t,r, r * t, r * (r * t), r * (r * t),.. .\}$.

pre-idempotent:

A type II FA groupoid $(A, *)$ is alleged to be *pre-idempotent* if $t * r$ is an idempotent in A for any $t,r \in A$. Note that if $(t, *)$ is an idempotent FA groupoid, at that point it is a pre-idempotent type II FA groupoid as fit. If $(t, *, g)$ is a leftoid, *i.e.*, $t * r := g(t)$ for some map $g : A \rightarrow A$, at that point $g(g(t)) = g(t)$ implies $(A, *, g)$ is a pre-idempotent type II FA groupoid.

Theorem 4.1: *Let* $(A, *)$ *be a* type II FA *groupoid. Let* $\lambda \in A$ *such that* $[t,r]_{p(q)} = \{t,r, \lambda, \lambda,...\}$ *for any* $t,r \in A$*. At that point* $(A, *)$ *is a* type II FA *pre-idempotent groupoid, and*

(i) *if* $p(q)= 0$*, at that point* $(r * t) * r = r * t$,

(ii) *if* $p(q)= 1$*, at that point* $(t * r) * r = t * r$,

(iii) *if* $p(q)= q$*, at that point* $b * (r * t)= r * t$,

(iv) *if* $p(q)=1 + c$*, at that point* $b * (t * r)= t * r$.

Proof:

(i) If $p(q)= 0$, at that point $[t,r]_{p(q)} = \{t,r, r * t, (r * t) * r, ((r * t) * r) * (r * t),.. .\}$. It surveys that $(r * t) * r = r * t$ and $r * t = ((r * t) * r) * (r * t)= (r * t) * (r * t)$, proving that $(A, *)$ is a type II FA pre-idempotent groupoid with $(r * t) * r = r * t$.

(ii) If $p(q) = 1$, at that point $[t,r]_{p(q)} = \{t,r, (t * r) * r, ((t * r) * r) * (t * r),.. .\}$. It surveys that $(t * r) * b = t * r$ and $t * r = ((t * r) * b) * (t * r)= (t * r) * (t * r)$, proving that $(A, *)$ is a type II FA pre-idempotent groupoid with $(t * r) * r = t * r$.

(iii) If $p(q)= q$, at that point $[t,r]_{p(q)} = \{t,r, r * t, b * (r * t), (r * (r * t)) * (r * t),.. .\}$. It surveys that $b * (r * t)= r * t$ and $r * t = (r * (r * t)) * (r * t)= (r * t) * (r * t)$, proving that $(A, *)$ is a type II FA pre-idempotent groupoid with $b * (r * t)= r * t$.

(iv) If $p(t = 1+ t$, at that point $[t,r]_{p(t)} = \{t,r, t * r, t *(t * r), (t * r) *(r *(t * r)),.. .\}$. It surveys that $b * (t * r)= t * r$ and $t * r = (t * r) * (r * (t * r)) = (t * r) * (t * r)$, proving that $(X, *)$ is a type II FA pre-idempotent groupoid with $b * (t * r)= t * r$.

Semi-lattice:

A type II FA groupoid $(A, *)$ is alleged to be a type II FA *semi-lattice* if it is idempotent, commutative and asso- ciative.

Theorem 4.2: *If* $(A, *)$ *is a* type II FA *semi-lattice and* $p(a) = p_0 + p_1a \in q2[[a]]$, *at that point there exists an element* $\lambda \in A$ *such that* $[t,r]_{p(a)} = \{t,r,\ \lambda,\ \lambda,..\ .\}$ *for any* $t,r \in A$.

Proof: If $(A, *)$ is a type II FA semi-lattice, at that point it is a type II FA pre-idempotent groupoid. Prearranged $t,r \in A$, we have $(r * t) * r = (t * r) * r = t * (r*r) = t * r$, $r * (r * t) = (r * t) * t = r * t$, $r * (t * r) = (r * t) * r = (t * r) * r = t * (r*r) = t * r$, $(t * r) * r = t * (r*r) = t * r$. If we take $\lambda := t * r$, at that point $[t,r]_{p(t)} = \{t,r,\ \lambda,\ \lambda,..\ .\}$ for any $t,r \in A$.

5. SMARANDACHE DISJOINTNESS

Proposition: *The class of Type II FA and the class of pre-idempotent groupoids are Smarandache disjoint.*

Proof:

Let $(A, *, 0)$ be mutually a *type II FA* and a type II FA pre-idempotent groupoid. At that point $t * r = (t * r) *(t * r) = 0$ and $r * t = (r * t) *(r * t) = 0$, by type II FA pre-idempotence and (D2), for any $t,r \in X$. By (D3), it surveys that $a = b$, which demonstrates that $|A| = 1$.

Proposition: *The class of groups and the class of* type II FA *pre-idempotent groupoids are Smarandache disjoint.*

Proof:

Let $(A, *, 0)$ be mutually a group and a type II FA pre-idempotent groupoid. At that point, for any $a \in A$, we have $a = t * e = (t * e) * (t * e) = t * t$. It surveys that $e = t * t^{-1} = (t * t) * t^{-1} = t * (t * t^{-1}) = t * e = a$, proving that $|A| = 1$. A type II FA groupoid $(A, *)$ is alleged to be an L^4-type II FA *groupoid* if, for all $t,r \in A$,

(L1) $((r * t) * r) * (r * t) = r * t$,

(L2) $(r * t) * ((r * t) * r) = (r * t) * r$.

Proposition: *Let* $(A, *)$ *be a* type II FA *groupoid and let* $p(t) \in q2[[t]]$ *such that* $p(t) = t4q(a)$ *for nearly* $q(t) \in q2[[a]]$. *If nearby exist* $\lambda, v \in A$ *such that* $[t,r]_{p(a)} = \{t,r,\ \lambda,\ v,\ \lambda,\ v,\ \alpha_1,\ \alpha_2,..\ .\}$, *where* $\alpha_i \in q2$ *for any* $t,r \in A$, *at that point* $(A, *)$ *is an* $L4$ -type II FA *groupoid*.

Proof:

Since $p(a) = a^4q(a)$, we have $\quad_{p(a)} = \{L, L, L, L, \alpha_1, \alpha_2,..\ .\}$, where $\alpha_i \in \{L, R\}$. It surveys that $[t,r]_{p(a)} = \{t,r,\ r*t,\ (r*t) *r,\ ((r*t) *r) *(r*t),\ (((r*t) *r) *(r*t)) *((r*t) *r),..\ .\}$. This expressions that $\lambda = r * t$, $v = (r * t) * r$, and hence $((r * t) *r) * (r * t) = r * t$ and $(r * t) * ((r * t) *r) = (r * t) * r$, evidencing the proposition.

Proposition: *Every* L^4 -type II FA *groupoid is* type II FA *pre-idempotent*.

Proof:

Given $t,r \in X$, we have $(r * t) * (r * t) = ((r * t) * b) * (r * t) = r * t$, proving the proposition.

Proposition: *The class of* L^4 -type II FA *groupoids and the class of groups are Smarandache dis- joint.*

Proof:

Let $(A, *)$ be both an L^4 -type II FA groupoid and a group with uniqueness e. At that point $((r * t) * r) * (r * t) = r * t$ for all $t,r \in A$. Subsequently any group has the cancellation laws, we obtain $(r * t) * r = e$. If we smear this to (L2), at that point we have $(r * t) * e = e$. This resources that $r * t = e$. It surveys that $e = r * t = ((r * t) * b) * (r * t) = (e * b) * e = b$, proving that $|A| = 1$.

Proposition: *The class of* L^4 -type II FA *groupoids and the class of BCK-algebras are Smaran- dache disjoint.*

Proof:

Let $(A, *)$ be both an L^4 -type II FA groupoid and a *BCK* -FA with a special element $0 \in A$. Given $t,r \in A$, we have $0 = 0 * a = (r*r) * a = (r * t) * b = (r * t) *((r * t) * b)= (r * t) *0= r * t$. Similarly, $t * r = 0$. Since X is a *BCK* -FA, $a = b$ for all $t,r \in X$, evidencing that $A = \{0\}$.

Let $p(x) \in q2[[t]]$. A groupoid $(A, *)$ is alleged to be a type II FA *Leonardo Pisanosemi-lattice* if for any

$t,r \in X$, nearby exists $\lambda = \lambda(t,r, p(t))$ in A reliant on $t,r, p(t)$ such that $[t,r]_{p(t)} = \{t,r, \lambda, \lambda,...\}$.

Note that every one type II FA Leonardo Pisanosemi-lattice is a type II FA pre-idempotent groupoid substantial one of the surroundings $(r * t) * r = t * r$, $r*(r * t)= r * t$, $r * (t * r)= t * r$, $(t * r) * r = t * r$ unconnectedly (and simultaneously).

Proposition: *Let* $(A, *)$ *be a t* type II FA *groupoid and let* $p(t) \in q2[[a]]$ *such that* $p(t)= t3q(t)$ *for approximately* $q(a) \in q2[[t]]$ *with* $q_0 = 1$. *At that point* $(A, *)$ *is a* type II FA *Leonardo Pisanosemi-lattice.*

Proof:

Meanwhile $p(a)= a^3q(a)$, where $q(a) \in q2[[a]]$ with $q_0 = 1$, we partake $\wedge_{p(t)} = \{L, L, L, R, \alpha_1,\alpha_2,...\}$, where $\alpha_i \in \{L, R\}$. If we let $\lambda := r * t$, $v := (r * t) * r$, at that point $[t,r]_{p(t)} = \{t,r, r * t = \lambda, (r * t) * r= v, ((r * t) * r) * (r * t)= v * \lambda = \lambda,...\}$. It surveys that $r * t = \lambda = v * \lambda = v = (r * t) * r$. Hence $[t,r]_{p(t)} = \{t,r, r * t, r * t,...\}$, evidencing the proposition. A type II FA groupoid $(A, *)$ is alleged to be an LRL^2-type II FA *groupoid* if for all $t,r \in A$, $(LRL^2 1)$ $(r * (r * t)) * (r * t)= r * t$, $(LRL^2 2)$ $(r * t) * (r * (r * t)) = r * (r * t)$.

Proposition: *Let* $(A, *)$ *be a* type II FA *groupoid and let* $p(a) \in q2[[a]]$ *such that* $p_0 = p_2 = p_3 = 0$, $p_1 = 1$. *At that point* $(A, *)$ *is an LRL*2 -type II FA *groupoid*.

Proof:

Since $p(t) \in q2[[t]]$ such that $p_0 = p_2 = p_3 = 0$, $p_1 = 1$, we partake $\wedge_{p(t)} = \{L, R, L, L, \alpha_1,\alpha_2,...\}$, where $\alpha_i \in \{L, R\}$. If we let $\lambda := r * t$, $v := r * (r * t)$, at that point $[t,r]_{p(t)} = \{t,r, r * t = \lambda, r * (r * t)= v, (r * (r * t)) * (r * t)= \lambda, [(r * (r * t)) * (r * t)] * [r * (r * t)],..\}$. It surveys that $(r *(r * t)) *(r * t)= r * t$ and $(r * t) *(r *(r * t)) = r *(r * t)$, evidencing the proposition.

Proposition: *The class of* LRL^2 -type II FA *groupoids and the class of groups are Smarandache disjoint*.

Proof:

Let $(A, *)$ be both an LRL^2 -type II FA groupoid and a group with a superior element $e \in A$. Given $t,r \in A$, we have $(r * (r * t)) * (r * t)= r * t$. Since every one group has cancellation laws, we attain $r * (r * t)= e$. It surveys that $r * t = (r * t) * e = (r * t) * (r * (r * t)) = r * (r * t)= e$, and henceforward $e = r * (r * t)= r * e = b$. This demonstrates that $|A| = 1$, verifying the proposition.

A type II FA groupoid $(A, *)$ is alleged to be an R^4-type II FA *groupoid* if for all $t,r \in A$, $(R^4 1)$ $(t * r) * (r * (t * r)) = t * r$, $(R^4 2)$ $(r* (t * r)) * (t * r)= r * (t * r)$.

Proposition: *Let* $(A, *)$ *be a* type II FA *groupoid and let* $p(a) \in q2[[a]]$ *such that* $p_0 = p_1 = p_2 = p_3 = 1$. *At that point* $(A, *)$ *is an R*4 -type II FA *groupoid.*

Proof:

Meanwhile $p(x) \in q2[[a]]$ such that $p_0 = p_1 = p_2 = p_3 = 1$, we partake $\wedge_{p(t)} = \{R, R, R, R, \alpha_1, \alpha_2, .. .\}$, where $\alpha_i \in \{L, R\}$. If we let $\lambda := t * r$, $v := b * (t * r)$, at that point $[t,r]_{p(t)} = \{t, r, t * r = \lambda, r * (t * r) = v, (t * r) * (r * (t * r)) = \lambda, [r * (t * r)] * [(t * r) * (r * (t * r))], .. .\}$. It surveys that $t * r = \lambda = \lambda * v = (t * r) * (r * (t * r))$ and $r * (t * r) = v = v * \lambda = (r * (t * r)) * (t * r)$, evidencing the proposition.

Theorem 5.1: *The class of* R^4 -type II FA *groupoids and the class of groups are Smarandache disjoint*.

Proof:

Let $(A, *)$ be mutually an R^4 -type II FA groupoid and a group with a superior element $e \in A$. Given $t,r \in A$, we have $(t * r) * (r * (t * r)) = t * r$. Since every one group has cancellation laws, we attain $b * (t * r) = e$. By put on $(R^4 2)$, we have $e * (t * r) = e$, *i.e.*, $t * r = e$. By $(R^4 1)$, $r = e * (r * e) = e$. This demonstrates that $|A| = 1$, evidencing the proposition.

Proposition: *Every implicative BCK-FA is an* R^4 -type II FA *groupoid*.

Proof:

If $(A, *, 0)$ is an implicative *BCK* -FA, at that point $t * (r * t) = t$ and $(t * r) * r = t * r$ for any $t,r \in A$. It surveys proximately that $(A, *)$ is an R^4 -type II FA groupoid.

6. APPLICATIONS OF SMARANDACHE TYPE II FA GROUPOID STRUCTURES.

Smarandache type II FA groupoid structures, such as Smarandache groupoids, Smarandache near-rings, and Smarandache semi-rings, have applications in automaton theory, error correcting codes, and the construction of S-sub-Y Automatons, which are discussed in this section. The semi-automaton and automaton are built using semigroups with a type II FA groupoid structure. We employ the generalized concept of type II FA groupoid semigroups in this section, resulting in the following. Smarandache type II FA groupoid semigroups are utilized to produce Smarandache semi-automaton and Smarandache automaton. As a result, the Smarandache type II FA groupoid can be used to build finite machines. Smarandache type II FA groupoid semi automata and Smarandache automaton employing Smarandache free groupoids are introduced here. The definition of Smarandache type II FA groupoid is the first step in this section.

Definition 6.1 : Let K1 and K2 be any two Smarandache type II FA groupoid mechanisms where K1 = (q1,Xs ,Ys ,ds ,ls) and K 2 = (q2, Xs ,Ys ,ds ,ls) with an supplementary assumption X2 = Y1.

The Smarandache type II FA groupoid automaton composition succession represented by of K1 and K 2 is defined as the succession alignment of the type II FA groupoid machine K1 = (q1, X1, Y1, d1, l1) and K2 =(q2, X2, Y2, d2, l2) with K1 ╫ K 2 = (q1 ´ q2, X1, Y2, d, l) where d ((q1, q2), X1) = (d1(q1, X1), d2 (q2, l1(q1, X1)) and l((q1, q2), X1) = (l2(q2, l1(q1, X1)) ((q1, q2) Î q1 ´ q2, X1 Î X1). This type II FA groupoid mechanism operates as follows: An input X1 Î X1 operates on q1 and rigidities a state conversion into q1' = d1(q1, X1) and an output Y1 = l1(q1, X2) Î Y1 = X2. This productivity Y1 operates on q2 renovates a q2 Î q2 into q2' = d2 (X2, Y1) and products the productivity l2 (q2, Y1).

At that point K1 ╫K 2 is in the next state (q1', q2') which is clear as of the succeeding circuit:

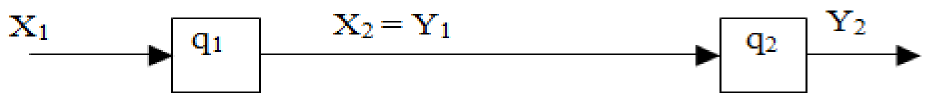

Figure 1. The smarandache type II FA groupoid automation composition operates of K1and K2.

The obvious question is whether we have a direct product that corresponds to the parallel composition of the two Smarandache type II FA groupoid automatons K1 and K 2. Visibly the Smarandache direct product of type II FA groupoid automatons K1 *K 2 since q1, and q2 can be construed as two parallel slabs. X1 operates on qi with output Yi (i Î {1,2}), X1* X2 operates on q1 *q2, the productivities are in Y1 *Y2.

Result:

The circuit is specified by the succeeding diagram:

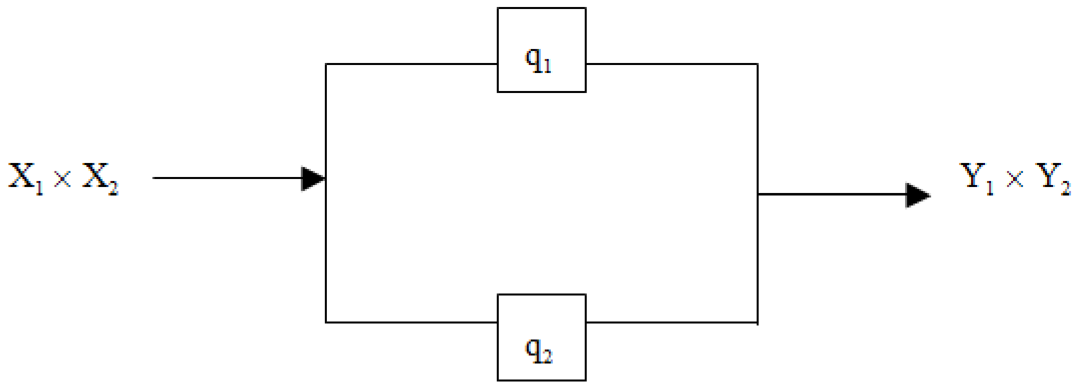

Figure 2. The smarandache direct product of type II FA groupoid automation composition operates of K1* K2.

7. CONCLUSIONS

I present the idea of generalized Leonardo Pisano over a type II FA groupoid in this work, and i analyze it in particular for the case when the type II FA groupoid encompasses type II FA idempotents and type II FA pre-idempotents. The concept of Smarandache-type II-FA was used to develop a number of relations on type II FA groupoids from generalised type II FA Leonardo Pisano. This finding can be used to apply type II FA groupoid theory in new ways. The results of this work will be useful in the creation of type II FA groups and their algebraic properties.

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